ANSWERS TO STUDY QUESTIONS

Chapter 17

17.1. The details are described in section 17.1.1.

17.3. Because of its declining payment pattern, a CAM would be most useful in an economy with persistent deflation where rents and land values decline over time. In an economy free of persistent deflation, the declining payment pattern in the CAM is an undesirable characteristic for many borrowers, and for many debt investors as well. From the borrower's perspective, it causes the initial loan payment to be excessively high, and does not well match the loan payment pattern with the likely income generation pattern of the underlying property that is being financed with the mortgage. From the perspective of the typical mortgage investor, the CAM likely requires an inconvenient (and possibly expensive) reinvestment of capital each period as the mortgage is amortized.

17.5. In a CPM, a "hybrid" payment pattern can be obtained by setting the maturity of the loan shorter than the amortization rate. This results in a balloon payment at maturity, but the size of the balloon can be reduced by increasing the payment level (faster amortization).

17.7. The adjustable rate mortgage (ARM) reduces the interest rate risk for the lender, making a lower interest rate possible while also improving the affordability of a mortgage for the borrower. The advantage of the ARM in reducing the initial interest rate is particularly strong for long-term mortgages during times when a steeply upward-sloping yield curve is prevailing in the bond market. Note that this does not imply that the borrower is getting something for nothing, for the borrower absorbs the interest rate risk that the lender avoids.

17.9. The effect of the discount points would be to increase the mortgage YTM over the stated contract interest rate in the loan.

17.11. The YTM from the lender's perspective at the time of loan origination is often referred to as the annual percentage rate, or APR.

17.13. a. The NPV of refinancing can be calculated as the DCF-based present values of the old loan minus the new loan, i.e., $PV(CF^{OLD}) - PV(CF^{NEW})$.

b. The new loan should be evaluated for an amount such that the actual cash disbursement the borrower receives is just sufficient to pay off the old loan exactly.

c. Defining the opportunity cost of capital as the yield on the new loan assures that $PV(CF^{NEW})$ equals the cash disbursement to the borrower on that loan. And the condition on the new loan amount requires that this cash disbursement must equal the amount required to pay off the old loan, OLB^{OLD}. These conditions imply that $PV(CF^{NEW}) = OLB^{OLD}$, and the NPV rule can be re-written as $P(CF^{OLD}) - OLB^{OLD}$. Thus, the refinancing decision is fundamentally not a comparison of two loans. The refinancing decision is simply a comparison of the current liability value of the old loan with the cash that would currently be required to pay off the old loan.

17.15. It is important to keep the loan amount constant between the old and new loan in a refinancing analysis in order to keep the amount of debt constant. This is to keep separate the refinancing decision from the decision to change the degree of leverage on the equity investment.

17.17. a. The fact that interest rates may fall further in the near future raises the possibility that the NPV of refinancing could be substantially greater in the near future than it is today. This possibility could make it worthwhile to wait.

b. The classical NPV decision criterion requires maximizing the NPV across all mutually exclusive alternatives. Refinancing today versus waiting and possibly refinancing in the future are mutually exclusive alternatives. The latter alternative may have the greater NPV, even evaluated at its present value today and considering the risk involved. If so, then one would choose to wait. Another possibility is that the market value of the old loan can be observed. Such a market value would incorporate the prepayment option value. In this case, the sign of the NPV defined as the old loan's current market value minus the cost to pay off the old loan today will correctly indicate whether refinancing is optimal.

17.19. a.

Month	OLB (Beg.)	PMT	INT	AMORT	OLB (End)
0					$2,000,000.00
1	$2,000,000.00	$13,333.33	$13,333.33	$0.00	$2,000,000.00
2	$2,000,000.00	$13,333.33	$13,333.33	$0.00	$2,000,000.00
3	$2,000,000.00	$13,333.33	$13,333.33	$0.00	$2,000,000.00

b.

Month	OLB (Beg.)	PMT	INT	AMORT	OLB (End)
0					$2,000,000.00
1	$2,000,000.00	$18,888.89	$13,333.33	$5,555.56	$1,994,444.44
2	$1,994,444.44	$18,851.85	$13,296.30	$5,555.56	$1,988,888.89
3	$1,988,888.89	$18,814.81	$13,259.26	$5,555.56	$1,983,333.33

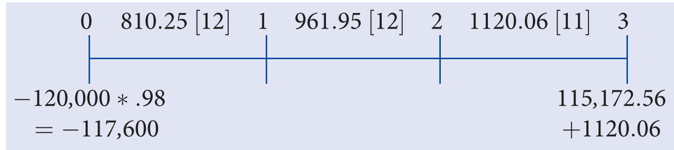
c.

Month	OLB (Beg.)	PMT	INT	AMORT	OLB (End)
0					$2,000,000.00
1	$2,000,000.00	$14,675.29	$13,333.33	$1,341.96	$1,998,658.04
2	$1,998,658.04	$14,675.29	$13,324.39	$1,350.90	$1,997,307.14
3	$1,997,307.14	$14,675.29	$13,315.38	$1,359.91	$1,995,947.23

17.21. a. $12,397.62
b. $15,001.12
The first payment is computed using the following formula:
2,000,000/*PV*(0.08/12,12,1) + 1.1 × *PV*(0.08/12,12,1)/(1 + 0.08/12)^12 + 1.1^2 × *PV*(0.08/12,336,1)/(1 + 0.08/12)^24

17.23. a. PMTs and OLB assuming a three-year holding period (maturity)

Year 1:	PV = 120000, N = 300, I/Y = 6.5, FV = 0 ⇒ **PMT** = −**810.25**
	Change N to 288 ⇒ PV = 118018.68
Year 2:	Contract Rate = min{6.25 + 2.50, 6.5 + 2.0, 6.5 + 4.0} = min{8.75, 8.5, 10.5} = 8.5%
	PV = 118108.68, N = 288, I/Y = 8.5, FV = 0 ⇒ **PMT** = −961.95
	Change N to 276 ⇒ PV = 116446.57
Year 3:	Contract Rate = min{8 + 2.5, 8.5 + 2.0, 6.5 + 4.0} = min{10.5, 10.5, 10.5} = 10.5%
	PV = 116446.57, N = 276, I/Y = 10.5, FV = 0 ⇒ **PMT** = −1120.06
	Change N to 264 ⇒ PV = **115172.56** = **OSB_{36}**

b. 

Using the cash flow keys on the financial calculator with:

C0 = −117,600, C1 = 810.25 [f1 or N_1 = 12], C2 = 961.95 [f2 or N_2 = 12], C3 = 1120.06 [f3 or N_3 = 11]

C4 = (1120.06 + 115172.56) [f4 or N_4 = 1], then compute **IRR** = **9.14%**

Two additional calculations of interest (not asked for in the question):

Initial Period Discount	= (Current Index + Margin) − Initial Rate
	= (4.75 + 2.50) − 6.50
	= 7.25 − 6.50
	= **0.75%** below market

The fact that the "current index + margin" exceeds the initial period contract rate implies that the initial rate is a "teaser," or below-market, rate.

The APR is the effective cost or yield on the loan under the following assumptions:

- The loan is held to maturity (i.e., the full amortization period).
- The index value is expected to remain at its current level forever.

Year 1: Payments are as calculated above: PMT = −810.25, OSB_{12} = 118018.68
Year 2: Contract rate assuming index value constant at 4.75% for all periods

= min{**4.75** + 2.50, 6.5 + 2.0, 6.5 + 4.0} = min{7.25, 8.5, 10.5} = 7.25%

PV = 118108.68, N = 288, I/Y = 7.25, FV = 0 ⇒ **PMT** = −**865.79**

***For the APR calculation, this is assumed to be the payment from month 13 until month 300 for this particular ARM.

Using the cash flow keys on the financial calculator with:

C0 = −117,600, C1 = 810.25 and f1 or N_1 = 12,

C2 = 865.79 and on the TI BAII+ , f2 = 288

but, on the HP10B, N_j has a maximum value of 99, hence you do the following:

C2 = 865.79, N_2 = 99, C3 = 865.79, N_3 = 99, C4 = 865.79, N_4 = 90
compute **IRR** = **7.40%**

17.25. a. 7.5% (no calculation required!)
b. 7.71%, with N = 360; i = 7.5%; PV = 2,000,000; FV = 0; compute PMT = 13,984.29; finally, change PV to 1,960,000 (to reflect points) just before computing i = 7.71%
c. 7.85%, with N = 360; i = 7.5%; PV = 2,000,000; FV = 0; compute PMT = 13,984.29; then change N to 96 and compute FV = 1,805,565; finally, change PV to 1,960,000 before computing i = 7.85%

17.27. a. $2,056,358. The 7.125% BEY equates to 7.02% MEY, as $12\{[(1 + .07125/2)^2]^{1/12} - 1\}$ = 7.02%. Applying this discount rate to the 96 monthly payments of $13,984.29 and the balloon of $1,805,565 gives a PV of $2,056,358.
b. The 7.875% BEY equates to 7.75% MEY, which gives the loan a PV of $1,971,401, which is $28,599 less than the contractual principal of $2,000,000. Thus, the loan requires 1.43 points (as 28,599/2,000,000 = 0.0143).

17.29. a. With the longer prepayment horizon, choose the lower-interest loan, as it has an effective rate of 6.36% versus 7.10% for the higher-interest loan.

b. With the shorter prepayment horizon choose the lower-points loan, as it has an effective interest rate of 6.69% versus 6.81% for the 6%/three-point loan.

17.31. **Method 1: "Brute Force" Approach**

Existing or Current or Old Loan

PV = 120,000, N = 360, I/Y = 9, FV = 0 ⇒ **PMT** = −**965.55**

Change N to 60, CPT FV = −115,056.16 [= OLB_{60}, the current outstanding loan balance and the amount that will be refinanced today assuming that refinancing costs are paid in cash]

Change N to 120, CPT FV = −107,315.69 [= OLB_{120}, the balance owing 5 years from today]

New Loan

PV = 115,056.16, N = 300, I/Y = 7, FV = 0 ⇒ **PMT** = −**813.19**

Change N to 60, CPT FV = −104,887.67 [= $OLB_{60,new}$, the balance owing in 5 years]

The benefit of refinancing is a reduction in monthly payments and a lower OLB in the future.

LOAN	**PMT**	**OLB** in 5 years
Existing	965.55	107,315.69
New	813.19	104,887.67
Difference	**152.36**	**2,428.02**

The cost is 2 points on the amount refinancing (i.e., the new loan balance). Call this TCP for transactions costs of prepayment. Then the NPV of refinancing is given by,

$$NPV_{ref} = -TCP + [PMT_{old} - PMT_{new}] * PVIFA\left(\frac{i_M}{12}, N\right) + \frac{OSB_{old,60+N} - OSB_{new,N}}{\left(1+\frac{i_M}{12}\right)^N}$$

Where i_M is the current market interest rate (7% in this question) and N is the expected remaining tenure in the home or remaining life in the mortgage.

Calculator keystrokes:

PMT = −152.36, N = 60, I/Y = 7, FV = −2,428.02 ⇒ PV = \$9,407.22
TCP = (0.02) ∗ (115,056.16 = 2,301.12
NPV = −2,301.12 + 9,407.22 =

Method 2: "Shortcut" or "Market Value versus Book Value" Approach

With this method we do not calculate payments on the new loan, so it is referred to as the "shortcut" method.

Go back and restart things by getting the PMT on the existing 9% loan staring at you in your calculator [PV = 120,000, N = 360, I/Y = 9, FV = 0 ⇒ PMT = −965.55]. Change N to 120 and compute FV = −107,315, the loan balance 5 years from today, OLB_{120}.

Calculate the PV of the remaining existing loan cash flows (60 PMTs of 965.55 + a lump sum of 107,315 in 5 years) payments at the current market rate of interest. That is, given the PMT and FV calculations above, change I/Y = 7 and compute **PV = \$124,463.23**.

This is the MARKET VALUE (or MV) of the existing loan − the present value of remaining loan CFs at the market rate of interest. The MV is higher than the OLB since interest rates have fallen (i.e., like bonds, mortgages sell at a premium if the market rate of interest exceeds the contract rate − most of the time anyway).

Subtract the balance owing on the existing loan − what we called OSB_{60} above − we will refer to this as the BOOK VALUE (or BV) of the loan − from the MV you just calculated to obtain the gain to refinancing in today's dollars (i.e., the PV of refinancing).

$$MV - BV = 124{,}463 - 115{,}056 = \$9{,}407$$

Subtract from this number the prepayment penalty (= 2% of OSB_{60}). The resulting number is the NPV of refinancing and is identical to the value we calculated in the brute force approach.

*17.33. a. *NPV* = \$6,048,772 − \$5,839,965 − \$150,000 − \$60,000 = −\$1,194. This is based on a current opportunity cost of capital (yield on the new loan) of 8.26% over a five-year maturity.

b. Yes, as the NPV would be positive including consideration of the prepayment option value in the old loan.

*17.35. The 8% MEY equates to 8.1345% BEY:

$$2\{[(1 + .08/12)^\wedge 12]^\wedge(1/2) - 1\}.$$

This is the yield the lender must be able to get on the loan's OLB invested for three years in the T-bonds. For every \$100 invested in the T-bonds, the lender will receive half the stated coupon rate (6%) every half year, thus, six semiannual payments of \$3 each, plus the par value of \$100 at the end. Discounting these payments at the required 8.1345% BEY yield (8.00% MEY) gives a PV of \$94.417557 for every \$100 invested. Thus (utilizing the homogeneity property of the present value equation), for every \$1 of OLB, the lender must be able to invest \$100/\$94.417557 = \$1.059125 into the T-bonds. Therefore, the required prepayment penalty is 5.9125 points. The 8%, \$1 million, 30-year-amortization mortgage would have an OLB after seven years of \$924,774. Thus, the required prepayment penalty would be 0.059125 × 924,774 = \$54,677. The total liquidating payment received by the lender would be \$924,774 + \$54,677 = \$979,451, which could be invested in the three-year, 6% T-bond to obtain an 8.1345% (BEY, or 8.00% MEY) yield on the \$924,774 OLB.