APPENDIX 30C

SIMPLE MODEL OF OPTIMAL LANDLORD SEARCH FOR TENANTS

This appendix presents a simple numerical example of optimal landlord behavior searching for tenants, in the form of a model of optimal asking rent. The analysis here may also be of interest as an example of how modern business quantitative analysis techniques can be used to shed light on basic commercial property management and investment decision issues. More generally, a model like this may be applied beyond the leasing decision, to shed some light, for example, on optimal behavior in the purchase or sale of investment property.[1] However, it is important to note that the model presented here is not an equilibrium model and considers only one side of what is, more realistically in most situations, a two-sided search market.

Consider the following simplified model of optimal asking rent for a landlord with an empty space. In particular, suppose the search problem is characterized by the following conditions:

1. Potential tenants "arrive" (or are found) randomly at an average rate of one per month. The expected wait time until the first potential tenant is found is one month, until the second is found is two months, etc.[2]
2. The ex ante probability distribution of the maximum rent each potential tenant will accept (the tenant's reservation rent) is a normal probability distribution with a mean of \$10/SF/year and a standard deviation of ±\$1/SF/year. All leases are for five-year lease terms, annual rent, payments at beginnings of years. The landlord only finds out what each tenant is willing to pay when that tenant "arrives."
3. If the tenant refuses the landlord's asking rent, the landlord has to wait until the next potential tenant arrives, and the space remains vacant during the wait time. When the space leases, it will always lease at the landlord's asking rent.[3]
4. When a lease expires, this process repeats (no renewals), in perpetuity.
5. The intralease discount rate is 8 percent; the interlease discount rate is 12 percent.

Given this situation, the question we want to answer is, What asking rent will maximize the present value of the building?

To answer this question we can specify the following analytical model. Let

A = Asking rent

$N(A;10,1)$ = Cumulative normal probability less than A when mean is 10 and STD is 1[4]

[1]Recall our discussion of asset valuation "noise" in section 12.2 of Chapter 12. Note also that the problem in this appendix is an example of optimal search theory as introduced more broadly in the Appendix 12A. This type of search model is also closely related to option value theory as described in Chapters 5 and 27.

[2]This is what is known as a Poisson arrival process. It is characterized by being "memoryless," that is, one arrival is not influenced by another arrival. Obviously, this is a simplification of the real world process by which landlords and tenants search for each other, but it does capture much of the essence of the reality. Mathematically, the probability that the first potential tenant will be found t periods from now is $\lambda_e{}^{-\lambda t}$, where e is the base of natural logs (approximately 2.7183) and l is the average rate of arrival per period. In a Poisson process, the expected time until the first arrival is the inverse of the arrival rate, $1/\lambda$, and the expected time until the nth arrival is n/λ. With time measured in years, condition (1) is postulating $\lambda = 12$ arrivals /year.

[3]Conditions (2) and (3) together represent the lease negotiation process. Once again, the model abstracts and simplifies from the real world, while still capturing much of the essence.

[4]For example, in Excel this quantity is given by the function NORMDIST(A,10,1,1).

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p = Probability tenant refuses landlord's offer = $N(A;10,1)$

w = Expected wait time (in years) until space is leased (average length of vacancy period between leases)

L = PV of each lease at time of signing (intralease discounting)

V = PV of building (perpetuity)

vac = Expected vacancy rate for building

Now consider the expected wait time, w. As p is the probability any one potential tenant will refuse the landlord's offer, $(1 - p)$ is the probability any one tenant will accept the offer. The expected time until the first tenant arrives is (1/12) years. Similarly (n/12) is the expected time (in years) until the nth potential tenant arrives. But the second potential tenant will never arrive if the first one accepts the landlord's offer. And the third tenant will never arrive if the second one accepts the landlord's offer. Consider the expression:

$$(n/12)(p^{n-1})(1-p) \tag{C.1}$$

This expression is the product of three factors: (n/12) is the expected time until the nth potential tenant arrives; p^{n-1} is the probability that all $n - 1$ previous arrivals have refused the landlord's offer (i.e., the probability that the landlord has to wait for the nth arrival); and $(1 - p)$ is then the probability that the nth arrival will accept the landlord's offer. The unconditional expected waiting time for the landlord, w, is thus a sum of expressions like (C.1), summed across all of the possible number of potential tenant arrivals, which is, in principle, infinite ($n = 1, 2, \ldots, \infty$). Therefore, the landlord's expected waiting time until she finds a tenant for a given space is

$$w = \frac{1}{12}(1-p) + \frac{2}{12}p(1-p) + \frac{3}{12}p^2(1-p) + \cdots \tag{C.2}$$

Equation (C.2) is an infinite series, but it has a simple finite value, namely,[5]

$$w = \frac{1}{(1-p)12} \tag{C.2a}$$

Lease value as a function of the asking rent, A, is given by the annuity formula (with payments in advance):

$$L = \frac{A(1.08)[1 - (1/1.08)^5]}{0.08} \tag{C.3}$$

The property value, V, is then given by the formula for the sum of an infinite geometric series, as a perpetuity of leases, each expected to occur $5 + w$ years after the previous lease was signed, discounting at the interlease discount rate between leases. The first lease is expected to be signed w years from time 0:[6]

$$V = \frac{L/(1.12)^w}{1 - (1/1.12)^{(w+5)}} \tag{C.4}$$

The average vacancy in the space is simply given by the expected time between leases divided by the lease term (five years):

$$vac = w/(w+5) \tag{C.5}$$

[5]There is some intuition in this result. A Poisson arrival process with arrival rate l in which each arrival has an independent probability $(1 - p)$ of being a "hit" is equivalent to a Poisson arrival process of hits with arrival rate $\lambda(1 - p)$. In a Poisson process, the expected time until the first arrival is the inverse of the arrival rate. Thus, the expected time until the arrival of the first hit is $1/[\lambda(1 - p)]$, or in the present case, $1/[12(1 - p)]$.

[6]Recall that the formula for an infinite geometric series is the first term in the series divided by the quantity one minus the common ratio. (See sections 8.2.1 and 8.2.7 in Chapter 8.) This is also how we derive formula (C.2a).

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This model can be easily solved quantitatively in a computer spreadsheet. Try some values of the asking rent, A, until you find the one that maximizes the present value of the space, V. Here is the result of such an iterative search procedure, starting at the average potential tenant reservation rent of $10/SF, and working up from there.

A	V
$10.00	$95.48
$11.00	$96.04
$12.00	$54.64
$10.90	$96.87
$10.80	$97.40
$10.70	$97.68
$10.60	$97.75
$10.50	$97.65

Thus, the optimal asking rent in this case is $10.60/SF (to the nearest dime), which gives a building value of $97.75/SF. At this asking rent, the expected waiting time to find a tenant that takes the rent (expected vacant period between leases) is $w = 0.304$ year, that is, between three and four months. This implies an optimal long-term average vacancy rate of $vac = 0.304/5.304 = 5.7$ percent.[7]

Now repeat this example only suppose the landlord's ex ante uncertainty surrounding the rent the potential tenants will take is doubled. That is, assume everything is the same except the standard deviation of the normal probability distribution is $2/SF instead of $1/SF. Thus, $p = N(A;10,2)$. Now we find that the optimal rent is $11.80/SF, giving a building value of $104.87/SF with an average vacant period of 0.453 year (about five months) and an average vacancy rate of 8.3 percent.

Note that the optimal asking rent, the optimal average vacancy rate, and the building value all increase with the uncertainty or range in the maximum rent the potential tenants are willing to accept. This is a general result. The two cases examined here are shown in the graph in Exhibit 30C-1. (The numbers on the left-hand vertical scale refer to $/SF for the top [solid] lines indicating property value, and this same scale refers to average percent vacancy for the bottom [dashed] lines. The triangle markers indicate the case with the lower rent uncertainty.) The general shapes of the curves in this graph are also a general result for typical realistic numbers.

From the previous analysis, we can derive several results about the optimal asking rent for the landlord in this simple model:

- Other things being equal, the optimal asking rent is higher the more uncertainty there is about the rental market.
- The greater is the rental market uncertainty, the more "forgiving" is the negative impact on property value due to an equal dollar magnitude error on the landlord's part in not selecting the optimal asking rent (i.e., the curve is "flatter" for higher variance in the rent distribution, at least on the upside).
- The effect on property value is relatively forgiving for erring on the side of asking too low a rent, while the negative value impact of asking too high a rent can be much more severe, especially when there is little uncertainty about the rental market.
- Other things being equal (such as the mean expected tenant reservation rent), and assuming optimal landlord behavior, the property is more valuable the greater is the uncertainty in the rental market, but this value effect is small even though our model

[7]At the $10.60 asking rent, the probability of potential tenant refusal is $p = N(10.60; 10,1) = 72.6\%$, which gives an acceptance probability of $1 - 0.726 = 0.274$. This gives an expected wait between leases of $w = 1/[(0.274)12] = 1/3.288 = 0.304$ year.

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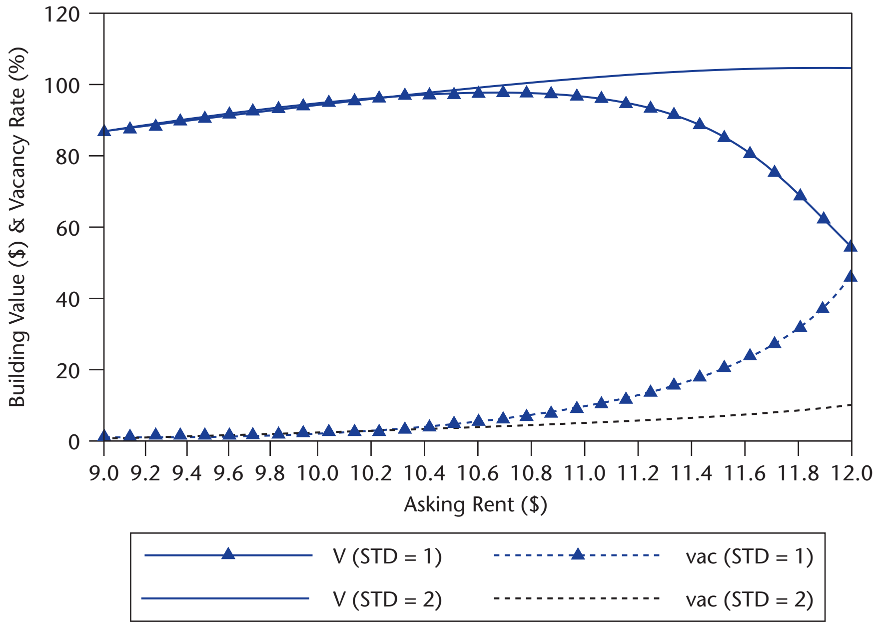
EXHIBIT 30C-1
Landlord Optimal Asking Rent and Vacancy Rate as a Function of Rental Market Uncertainty (Assumptions: Tenant maximum rent distribution normal with mean = $10/SF, lease term = five years, intralease discount rate = 8 percent, interlease discount rate = 12 percent.)



ignores the effect of rent uncertainty on the interlease discount rate (which might dampen or reverse this result).

Although the model on which these conclusions are based is a simplification of reality, the first three conclusions are fairly robust if one interprets them broadly or "figuratively." For example, they can be paraphrased in the following two more general principles of optimal tenant search and leasing strategy for a landlord:

1. Be a bit daring and aggressive in pursuit of good leasing deals if you have a lot of uncertainty about the rental market. (This is a generalization of the first two preceding points.)
2. Be conservative and play it safe if the landlord is very risk averse or if the rental market is very obvious, with little uncertainty about market rents. (This is a generalization of the third previous point.)

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