APPENDIX 29

EVALUATING MULTIPHASE DEVELOPMENTS: A REAL OPTIONS APPROACH

In this section, we will present a simple numerical example of a multiphase development project that illustrates all of the major option valuation procedures described in Chapter 27 as well as the compound option model of sequential construction phases described in section 29.4 of the Chapter 29 printed text. We will also show how the option model results compare to traditional DCF procedures used to evaluate development projects. An Excel® spreadsheet file also on this CD presents the detailed formulas used in this example and can serve as a basic template users may customize for their own applications.

Roth Harbor is a strategically and scenically located former brown-field site on the shore near the center of Wheatonville, Maine. Wheatonville is a former shipbuilding city now booming with high-tech startups and an influx of young professionals and empty-nesters, creating a serious housing shortage. The 50 acres of former industrial and warehouse property in Roth Harbor are currently zoned to allow 500 market-rate apartments to be developed. The property is owned by Ciochetti Enterprises LLC (CEC), which has plans to build the 500 units all at once in a single project, to be called Rentleg Gardens. But the Planning Commission of Wheatonville has a better idea.

Before we consider the Planning Commission's idea, however, let's take a look at the Rentleg Gardens project. Current apartment rents in Wheatonville suggest that the Rentleg Gardens apartments could charge gross rents of $1100/mo. with operating expenses of $6533/yr. per occupied unit, and average vacancy of 4 percent. Cap rates (y_V) on such properties are currently 8 percent. If the 500 Rentleg Gardens units existed today, the property would be worth:

$$[(1100 \times 12 - 6533) \times 0.96/0.08] \times 500 = \$40{,}000{,}000$$

based on a projected current NOI of $3,200,000/yr. and an average unit value of $80,000. Construction cost as of today would be $32 million, with a projected deterministic (riskless) growth rate of 2%/yr. Construction would take one year [implying a bill due next year of: (1.02)$32 million = $32.64 M]. (Environmental cleanup of the site has already been done by CEC, and the site is now ready for development.)

The Planning Commission's "better idea" has been dubbed *Roth Harbor Place* (RHP). The idea is that the Planning Commission would approve a "Special Zoning Exemption" that would allow much greater density, in a two-phase development. In return, the landowner would commit to:

- Provide mixed-income housing (approximately 25 percent of units below market rent).
- Start construction on Phase I ("Frenchman Cove") no later than three years from now.
- Start construction on Phase II ("Fisher Landing") no later than five years from now.

Phase II cannot be started until Phase I is complete, and if Phase I is not started within three years, the Special Exemption expires and the land reverts to its previous as-of-right value based on a project like Rentleg Gardens.

The program of the RHP project is summarized in the decision tree shown in Exhibit 29A-1. In fact, this is a simplified tree. A more complete representation would allow, in essence, that the decisions represented in Exhibit 29A-1 could be taken anytime during the first

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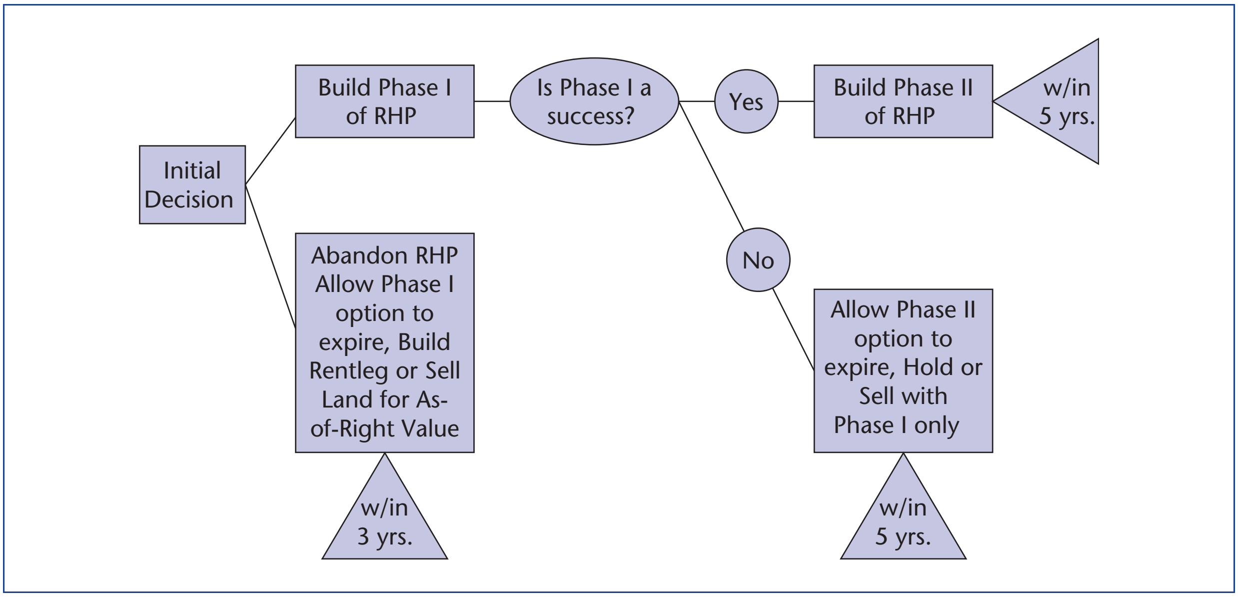

EXHIBIT 29A-1 Decision Tree Representation of the Roth Harbor Place Project

three years in the case of the decision to build Phase I, or anytime between the completion of Phase I and the end of year 5 in the case of the decision to build Phase II. Across time between each of the decision nodes that are under the control of the landowner (rectangular boxes) there occurs an outcome node (circles) that is not under the landowner's control that reflects the movements in the market for apartments in Wheatonville. Our real options based methodology will allow the consideration of all of these possibilities. Note also that if Phase I is built but the Phase II option expires unexercised, then the value of the property is based purely on the Phase I build-out, without any further residual land value, as the Phase I build-out already exceeds the density allowance in the as-of-right zoning.[1]

The economics of the Roth Harbor Place proposal are summarized as follows:

- Phase I (Frenchman Cove):
  - 900 units, at today's rents NOI = \$4,800,000/year.
  - At today's cap rate of $y_V = 8\%$. Thus:
  - Current value $V_0 = \$60,000,000$.
  - Construction cost as of today would be $K_0 = \$48,000,000$, and time-to-build is two years.
- Phase II (Fisher Landing):
  - 1600 units, at today's rents NOI = \$8,000,000/year.
  - At today's cap rate of $y_V = 8\%$. Thus:
  - Current value $V_0 = \$100,000,000$.
  - Construction cost as of today would be $K_0 = \$80,000,000$, and time-to-build is two years.

[1]If this were not the case, then the terminal nodes on each branch, indicated by the triangles, would include the "abandonment" value of any residual as-of-right development option that remains beyond whatever phases have been completed within each branch. Given the nature of the Special Zoning Exemption in the RHP project, such an abandonment option (Rentleg Gardens) exists only prior to construction of the first phase (Frenchman Cove).

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- In both cases (as also with Rentleg Gardens):
  - Market OCC for stabilized apartments (plus small lease-up risk premium) = 9%/year, a 5% risk premium over the risk-free rate of $r_f = 4\%$.
  - Growth in construction costs equals inflation: $g_K = 2\%$/year (riskless).
  - Volatility of built property = 15%/year.

To evaluate a project like RHP on a rigorous economic basis, considering opportunity cost and market equilibrium, we must model the project as a compound option. Because Phase II depends on Phase I, and neither phase is absolutely required to be begun at a prespecified time, the project involves an option on an option. There is also an "abandonment option" in the form of the as-of-right Rentleg Gardens development, which can be exercised at any time as an alternative to the first phase (Frenchman Cove) project.

In general with option modeling, we go to the end first, and then work backward in time. The possible end results in the RHP program are indicated by the triangles in Exhibit 29A-1. The project could end with Phase II constructed after Phase I, or with only Phase I constructed, or with abandonment of RHP and only the option to build Rentleg Gardens remaining. Because the abandonment option has no expiration time, we will need to evaluate it as of all possible future states of the world during the period when the overall RHP project can be undertaken. Therefore, a logical first step in the analysis of the RHP project is to evaluate its abandonment value, the as-of-right (perpetual) option to develop Rentleg Gardens.

In fact, it makes sense to evaluate the as-of-right option before we evaluate the RHP project, because it is not only the abandonment option of the RHP project, but also represents the opportunity cost of that special project. Because the as-of-right option is a simple perpetual option, we can evaluate it in any state of the world using the Samuelson-McKean Formula described in Chapter 27. Thus, we can evaluate it directly as of time 0. This value suggests how much the city might have to pay the current landowner (i.e., it suggests a fair market value of the land without the Special Zoning), for example, if the city wants to take over the site so it can open up the selection of a developer to other parties besides CEC. More fundamentally from an economic perspective, as noted, the abandonment option represents the opportunity cost of the RHP project, the basic benchmark necessary to compute the NPV of the Planning Commission's proposal. Furthermore, if the as-of-right project value currently equals or exceeds its Samuelson-McKean "hurdle value" (V^*), this suggests urgency in getting the alternative RHP project to supersede Rentleg Gardens, as the landowner should optimally immediately proceed with the as-of-right development in the absence of the Special Zoning Exemption. Recognition of such urgency could be important in the policy decision-making process.

Since the landowner has the right without obligation to build the as-of-right project (Rentleg Gardens) anytime, without limit, we evaluate the land by applying the Samuelson-McKean Formula for the value of a perpetual American call option, with Rentleg Gardens as the underlying asset and with time-to-build equal to one year.[2]

The calculations are shown below. First, the elements of the option valuation formula are determined:

$$
\begin{aligned}
&y_V = 8\%, y_K = (1+r_f)/(1+g_K) - 1 = 1.96\%, \sigma = 15\%, K_0 = \$32, V_0 = \$40,\\
\Rightarrow\ &\eta = \{y_V - y_K + \sigma^2/2 + [(y_K - y_V - \sigma^2/2)^2 + 2y_K\sigma^2]^{1/2}\}/\sigma^2\\
&\quad = \{.08 - .0196 + .15^2/2 + [(.0196 - .08 - .15^2/2)^2 + 2(.0196).15^2]^{1/2}\}/.15^2\\
&\quad = 6.63\\
&V^* = K_0(1+g_K)/(1+r_f)[\eta/(\eta-1)] = \$31.38[6.63/(6.63-1)]\\
&\quad = \$31.38(1.178) = \$36.96
\end{aligned}
$$

[2]This formula is presented in section 27.5 of Chapter 27, with the time-to-build extension described in Appendix 27.

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These elements are then applied in the Samuelson-McKean Formula with one year time-to-build:

$$C = \begin{cases} (V^* - K/(1+y_K))\left(\dfrac{V/(1+y_V)}{V^*}\right)^{\eta}, \text{ if } V/(1+y_V) \leq V^* \\ V/(1+y_V) - K/(1+y_K), \ \text{otherwise} \end{cases}$$

In this case, we have:

$$V/(1+y_V) = \$40/1.08 = \$37.04 > \$36.96 = V^*$$

which makes the as-of-right project worth:

$$\begin{aligned} C &= V/(1+y_V) - K/(1+y_K) = 40/1.08 - 32/1.0196 \\ &= \$37.04 - \$31.38 = \$5.65 \text{ million} \end{aligned}$$

The as-of-right land value of the Roth Harbor site (based on Rentleg Gardens) is therefore \$5.65 million. Furthermore, Rentleg Gardens exceeds its hurdle and so is ripe for immediate development. This means that there is some urgency in dealing with the current owner, CEC.

Step two in our evaluation of the RHP project is to develop a binomial tree of the as-of-right land values in each future state of the world relevant for evaluating the RHP project, to provide the "abandonment value" contingencies in the project analysis. This is done by applying the preceding Samuelson-McKean Formula to $V_{i,j}$ and K_j in each i, j of a binomial tree, using the same basic structure as we described in Appendix 27.[3] The result is shown in Exhibit 29A-2, for seven years of (annual) projections of V, K, and C based on Rentleg Gardens.[4]

Step three in evaluating the RHP project is to compute the value of the option to build Phase II (*Fisher Landing*). This is done in three steps:

- Build the Fisher Landing underlying asset value tree forward in time, through Year 7 (the latest that phase can be obtained, if the option is exercised at its expiration time in Year 5, given the two-year time-to-build requirement), starting from time 0 where $V_0 = \$100$M.
- Build the corresponding Fisher Landing construction cost tree forward in time, through Year 7, starting from $K_0 = \$80$ million.
- Then build the call option value tree through Year 5 (option expiration), working backwards from Year 5 to time 0. Exercise of the option (development of the project) obtains the completed Fisher Landing property two years after exercise.[5]

The result of these three steps is depicted in the binomial trees shown in Exhibit 29A-3.

By way of review from Chapter 27, let us walk through the process of deriving the numbers in Exhibit 29A-3. First, recall from Appendix 27 that you build the underlying asset value tree left to right forward in time to the expiration date, starting from the present

[3]The calculations underlying all of the exhibits in this section can be found in the accompanying Excel file.

[4]Note that to evaluate the RHP project we need only six periods in the as-of-right option (abandonment) value tree, because the Special Zoning Exemption expires after Year 5.

[5]The Fisher Landing option cannot be obtained prior to Year 2, the earliest possible completion date of Phase I. Thus, we won't end up using the Year 0 and Year 1 values of this option, but we might as well calculate them anyway. Note also that because Fisher Landing is the last phase of the overall project (no further subsequent development dependent on the completion of Phase II is possible), this is a simple call option of the type presented in Chapter 27. Furthermore, because the Phase II option does not even exist unless and until Phase I is complete, and given that Phase I already exceeds the as-of-right (RHP abandonment) build-out of the site, the terminal value of the Phase II option (at its expiration date) is simply: *Max*[Phase II Devlpt PV, 0]. If, contrary to what we have posited here, there were residual development rights on the part of the Roth Harbor site where Phase II would be built, then the terminal value of the Phase II option would be *Max*[Phase II Devlpt. PV, Resid. Devlpt. Rights Value]. (Such residual development rights could be quantified using the Samuelson-McKean Formula.)

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Year ("*j*"):	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Rentleg Gardens Expected Values:		**$40.37**	**$40.74**	**$41.12**	**$41.50**	**$41.89**	**$42.27**	**$42.67**
"down" moves ("*i*"). *Rentleg Gardens Value Tree* (as if new, ex-dividend):								
0	40.00	42.59	45.35	48.29	51.42	54.76	58.30	62.08
1		32.21	34.29	36.52	38.88	41.40	44.09	46.94
2			25.93	27.61	29.40	31.31	33.34	35.50
3				20.88	22.23	23.67	25.21	26.84
4					16.81	17.90	19.06	20.30
5						13.53	14.41	15.35
6							10.90	11.60
7								8.77
Year ("*j*"):	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
"down" moves ("*i*"). *Rentleg Construction Cost Tree*:								
0	32.00	32.64	33.29	33.96	34.64	35.33	36.04	36.76
1		32.64	33.29	33.96	34.64	35.33	36.04	36.76
2			33.29	33.96	34.64	35.33	36.04	36.76
3				33.96	34.64	35.33	36.04	36.76
4					34.64	35.33	36.04	36.76
5						35.33	36.04	36.76
6							36.04	36.76
7								36.76
Year ("*j*"):	**0**	**1**	**2**	**3**	**4**	**5**	**6**	
"down" moves ("*i*"). *Rentleg Land Value Tree* (Samuelson-McKean, reflecting 1 yr. time-to-build):								
0	5.65	7.43	9.34	11.41	13.64	16.05	18.64	
1		1.20	1.63	2.21	3.00	4.07	5.52	
2			0.26	0.35	0.47	0.64	0.86	
3				0.05	0.07	0.10	0.14	
4					0.01	0.02	0.02	
5						0.00	0.00	
6							0.00	

EXHIBIT 29A-2 Binomial Trees Depicting As-of-Right (Rentleg Gardens) Underlying Asset, Construction Cost, and Resulting Perpetual Option Valuation

(time 0). Then you work right to left backward in time to value the option at each node (*state of the world*) in the binomial tree. As an example, here are the calculations to value the underlying asset value tree for the two nodes (the two possible "states of the world") one year after time 0, and to project the construction cost for that year:

$$V_{0,1} = uV_{0.0}/(1+y_V) = (1+\sigma\sqrt{T/n})V_t/(1+y_V) = (1.15)\$100/(1.08) = \$106.48$$
$$V_{1,1} = dV_{0.0}/(1+y_V) = (1/u)V_{0.0}/(1+y_V) = (1/1.15)\$100/(1.08) = \$80.52$$
$$K_1 \quad = (1+r_f)K_0/(1+y_K) = (1+g_K)K_0 = (1.02)\$80 = \$81.60$$

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Year ("*j*"):	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Fisher Landing Expected Values:		**\$100.93**	**\$101.86**	**\$102.80**	**\$103.76**	**\$104.72**	**\$105.69**	**\$106.66**
"down" moves ("*i*"). *Fisher Landing Underlying Asset Value Tree* (as if new, ex-dividend):								
0	100.00	106.48	113.38	120.73	128.56	136.89	145.76	155.21
1		80.52	85.73	91.29	97.21	103.51	110.22	117.36
2			64.83	69.03	73.50	78.27	83.34	88.74
3				52.20	55.58	59.18	63.02	67.10
4					42.03	44.75	47.65	50.74
5						33.84	36.03	38.37
6							27.24	29.01
7								21.94
Year ("*j*"):	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
"down" moves ("*i*"). *Fisher Landing Construction Cost Tree*:								
0	80.00	81.60	83.23	84.90	86.59	88.33	90.09	91.89
1		81.60	83.23	84.90	86.59	88.33	90.09	91.89
2			83.23	84.90	86.59	88.33	90.09	91.89
3				84.90	86.59	88.33	90.09	91.89
4					86.59	88.33	90.09	91.89
5						88.33	90.09	91.89
6							90.09	91.89
7								91.89

Value of Option on Phase II (Fisher Landing), reflecting 2-yr. time-to-build:						
Year ("*j*"):	**0**	**1**	**2**	**3**	**4**	**5**
"down" moves ("*i*"):						
0	8.78	12.80	17.15	21.85	26.92	32.40
1		0.44	0.75	1.29	2.21	3.78
2			0.00	0.00	0.00	0.00
3				0.00	0.00	0.00
4					0.00	0.00
5						0.00

EXHIBIT 29A-3 Binomial Trees Depicting Phase II (Fisher Landing) Underlying Asset, Construction Cost, and Resulting Option Valuation with Two-Year Time-to-Build

And here is the binomial "*up*" jump probability calculation:

$$p = \frac{(1+r_V) - 1/(1+\sigma\sqrt{T/n})}{(1+\sigma\sqrt{T/n}) - 1/(1+\sigma\sqrt{T/n})} = \frac{1.09 - 1/1.15}{1.15 - 1/1.15} = 0.786$$

(Therefore, obviously, the probability of the "*down*" movement in underlying asset value is $1 - 0.786 = 0.214$.) Graphically, this can be represented by the "tree" shown in Exhibit 29A-4.

Recalling the basic binomial option value formula from Chapter 27 (the certainty-equivalence DCF formula from Chapter 10 Appendix 10C):

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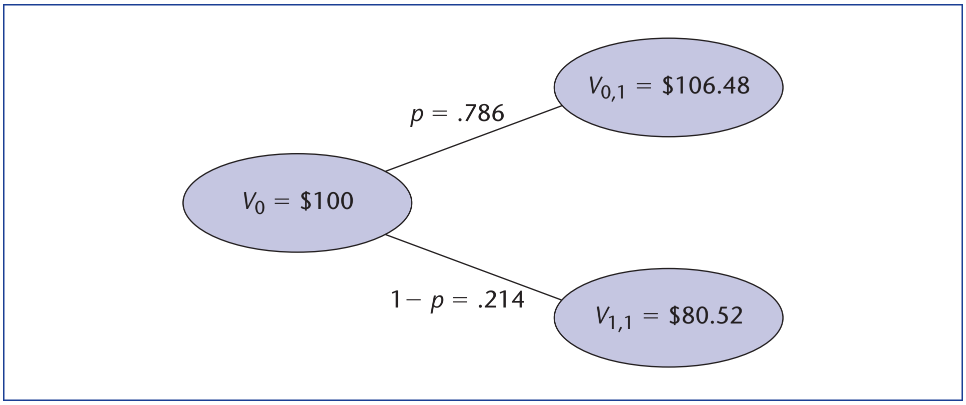

EXHIBIT 29A-4 Decision Tree Representation of the Underlying Asset Value for the Fisher Landing Project

$$C_{i,j} = Max\left\{\frac{V_{i,j}}{(1+y_V)^2} - \frac{K_{i,j}}{(1+y_K)^2}, \frac{(pC_{i,j+1} + (1-p)C_{i+1,j+1}) - (C_{i,j+1} - C_{i+1,j+1})\left[\frac{r_V - r_f}{(1+\sigma\sqrt{T/n}) - 1/(1+\sigma\sqrt{T/n})}\right]}{1+r_f}\right\}$$

We have, for example, the following last step of the valuation of the option at time 0:

$$C_{0,0} = Max\left\{\frac{100}{(1.08)^2} - \frac{80}{(1.0196)^2}, \frac{((.786)12.80 + (.214)0.44) - (12.80 - 0.44)\left[\frac{9\% - 4\%}{(1.15) - 1/(1.15)}\right]}{1.04}\right\}$$
$$= Max\{\$8.78, \$7.65\} = \$8.78$$

Note that the present value of the option to build Phase II alone would be \$8.78 million as of time 0. However, it is impossible to obtain the Phase II option without first developing Phase I, which itself will take two years. Thus, the value of \$8.78 million is not in itself relevant to valuing the RHP project. But the construction of this binomial value tree is a step along the way in our valuation of the overall project.

Step four in the valuation of the RHP project is to compute the value of the compound option to build Phase I, which obtains both *Frenchman Cove* and also the option to build *Fisher Landing*. This involves four subsidiary steps in itself:

- First, discount the Phase II option value we just calculated back two years in time, to obtain its present value in each period as a part of the underlying asset for the Phase I compound option, reflecting the Phase I time-to-build, which is two years. In other words, if you exercise the Phase I option, you will get the Phase II option only after a two-year lag. For purposes of providing the Phase II option component of the current underlying asset value for the Phase I option, we therefore need to know the present of the Phase II option as of each time when the Phase I option may be exercised, which is the future value of the Phase II option two years hence, discounted back two years.
- Then, develop the other part of the Phase I option's underlying asset value by building the Frenchman Cove value tree forward in time, through Year 5 (the last year it could be obtained, as the Phase I option expires in Year 3 and the project takes two years to build), starting from V_0 = \$60 million.
- Build the corresponding Frenchman Cove construction cost tree forward in time, through Year 5, starting from K_0 = \$48 million.
- Finally, build the Phase I compound call option value tree working backwards in time from Year 3 (option expiration) to time 0. Exercise of the option (development of Frenchman Cove) obtains the completed Frenchman Cove property, plus the Phase II option, both two years after exercise of the Phase I option (commencement of

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PV of 1 period delayed receipt of Phase 2 option value:

Year ("*j*"):	0	1	2	3	4
"down" moves ("*i*"):					
0	7.65	10.30	13.25	16.57	20.36
1		0.44	0.75	1.29	2.21
2			0.00	0.00	0.00
3				0.00	0.00
4					0.00

PV of 2 period delayed receipt of Phase 2 option value:

Year ("*j*"):	0	1	2	3
"down" moves ("*i*"):				
0	6.19	8.03	10.17	12.73
1		0.44	0.75	1.29
2			0.00	0.00
3				0.00

EXHIBIT 29A-5 Binomial Trees Depicting Discounting of Phase II (Fisher Landing) Option Value, One Year at a Time

Frenchman Cove construction). Abandonment for as-of-right land value is always an alternative to either holding or exercising the Phase I option.

Once again, let us walk through this part of the analysis one step at a time. First, we need to discount the Phase II option values back two years in time. This can be done one year at a time. The top panel in Exhibit 29A-5 shows a binomial value tree with values of the Phase II option discounted one year. This table is calculated right to left, working backwards in time, based on the Phase II current option values shown in the bottom panel of Exhibit 29A-3. We have, for example, the following certainty-equivalence DCF calculation for the present value of the Phase II option discounted one year in the *up* node of Year 1 (cell 0,1):[6]

$$PV_{0,1}[C_2] = \frac{((.786)17.15 + (.214)0.75) - (17.15 - 0.75)\left[\dfrac{9\% - 4\%}{(1.15) - 1/(1.15)}\right]}{1.04} = \$10.30$$

Now, similarly referencing the two possible subsequent values of the Phase II option discounted one year as shown in the top panel of Exhibit 29A-5, we have the following calculation of the Phase II option value discounted *two years,* as of time 0, as seen in the bottom panel of Exhibit 29A-5:

$$PV_0[C_2] = PV_{0,0}[PV_1[C_2]] = \frac{((.786)10.30 + (.214)0.44) - (10.30 - 0.44)\left[\dfrac{9\% - 4\%}{(1.15) - 1/(1.15)}\right]}{1.04} = \$6.19$$

Thus, as of time 0 the two-year *forward value* of the Phase II option is \$6.19 million. This value could be obtained as of time 0 by the exercise at that time of the Phase I option,

[6]Referencing nodes 0,2 and 1,2 in the bottom panel of Exhibit 29A-3, we see that the current value of the Phase II option in Year 2 will either be \$17.15 million, or \$0.75 million, depending on whether the underlying asset value moves up or down from the 0,1 state of the world in Year 1.

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Year ("*j*"):	**0**	**1**	**2**	**3**	**4**	**5**
Frenchman Cove Expected Values:		**$60.56**	**$61.12**	**$61.68**	**$62.25**	**$62.83**
"down" moves ("*i*"). *Frenchman Cove Underlying Asset Value Tree* (as if new, ex-dividend):						
0	60.00	63.89	68.03	72.44	77.13	82.13
1		48.31	51.44	54.77	58.32	62.10
2			38.90	41.42	44.10	46.96
3				31.32	33.35	35.51
4					25.22	26.85
5						20.30
Year ("*j*"):	**0**	**1**	**2**	**3**	**4**	**5**
"down" moves ("*i*"). *Frenchman Cove Construction Cost Tree*:						
0	48.00	48.96	49.94	50.94	51.96	53.00
1		48.96	49.94	50.94	51.96	53.00
2			49.94	50.94	51.96	53.00
3				50.94	51.96	53.00
4					51.96	53.00
5						53.00

EXHIBIT 29A-6 Binomial Trees Depicting Phase I (Frenchman Cove) Underlying Asset Completed Value and Construction Cost

that is, by the irrevocable commitment to build Phase I, which will result two years later in the completion of Phase I which obtains the option to build Phase II.

The two-year forward value of the Phase II option is, of course, only part of what the developer obtains upon the exercise of the Phase I option. He also obtains the net present value of the Frenchman Cove development project, the NPV_0 value for that phase, as we defined this term in section 29.1.1 of the printed text. Given the two-year construction period, this value can be represented as follows:

$$PV_t[V_{t+2} - K_{t+2}] = \frac{E_t[V_{t+2}]}{(1+r_V)^2} - \frac{K_{t+2}}{(1+r_f)^2} = \frac{V_t}{(1+y_V)^2} - \frac{K_t}{(1+y_K)^2}$$

To quantify this part of the Phase I value, we need to build the Frenchman Cove underlying asset value tree, and project the Frenchman Cove construction cost, starting from time 0 and building forward to a point two years (the time-to-build) beyond the expiration of the Phase I option in three years. This is shown in Exhibit 29A-6. Note that the value of the completed project grows from a current value of $60 million if the project was operational at time 0, to an expected value of $62.83 million in five years, even as the projected construction costs (as if due on completion) grow from $48 million to $53 million (that is, 2 percent per year).

The value of the Phase I option is then calculated working backwards in time from its expiration in Year 3. The value in Year 3 is the maximum of either:

(i) The as-of-right land value (land value based on the Rentleg Gardens project, which is the "abandonment value" of the RHP project); or

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Value of Option on Phase I (Frenchman Cove), reflecting 2-yr. time-to-build:

Year ("*j*"):	0	1	2	3
"down" moves ("*i*"):				Opt Expires
0	11.46	15.71	20.45	25.84
1		1.20	1.63	2.21
2			0.26	0.35
3				0.05

EXHIBIT 29A-7 Binomial Tree Depicting Phase I Option Value (Frenchman Cove + Phase II Option, with As-of-Right Abandonment Value)

(ii) The value of immediate exercise of the Phase I option (which obtains the completed Frenchman Cove project plus the Phase II option, both two years later):

This can be expressed as:

$$C_3 = Max\{As\text{-}of\text{-}right\ Land\ Value_3,\ PV_3[V_5 - K_5] + PV_3[Ph.II\ Opt_5]\}$$

The value in any earlier year is the current value of the maximum of either of the above two alternatives (i) and (ii) plus the third alternative of holding the "live" option unexercised for at least one more year:[7]

$$C_t = Max\{As\text{-}of\text{-}right\ Land\ Value_t,\ PV_t[V_{t+2} - K_{t+2}] + PV_t[Ph.II\ Opt_{t+2}],\ PV_t[C_{t+1}]\}$$

The result is the Phase I option value tree presented in Exhibit 29A-7. We see that as of the present (time 0), the Phase I option is worth \$11.46 million:[8]

$$C_{0,0} = Max\left\{5.65, \frac{60}{(1.08)^2} - \frac{48}{(1.0196)^2} + 6.19, \frac{((.786)15.71 + (.214)1.20) - (15.71 - 1.20)\left[\dfrac{9\% - 4\%}{(1.15) - 1/(1.15)}\right]}{1.04}\right\}$$
$$= Max\{\$5.65,\ \$11.46,\ \$9.63\} = \$11.46$$

As we have defined the Phase I option here, this value is in fact the present value of the Roth Harbor land with the Special Zoning Exemption, in other words, the gross value of the RHP project option. The \$11.46 million value includes consideration of the abandonment value (the perpetual option on Rentleg Gardens), the value of the Phase I project (including both the Frenchman Cove buildings and the compound option on the right to build Phase II), and the value of the option to wait and invest later in the Phase I project (including its option on Phase II).

Since the value of the site with the proposed Special Zoning Exemption for the two-phase RHP project is \$11.46 million, the *incremental value* added over the preexisting land

[7] Since the option includes the right to sell the land, and the present value today of the right to sell the land next period equals the Samuelson-McKean present value of the land today, the value of holding the option is the *maximum* of either the certainty-equivalent option valuation or the current Samuelson-McKean land value. However, the future sell-out option in the *maximum* function in the option value is redundant with the ability to sell the land directly today, which also exists and is already accounted for in the first term of the *maximum* function, and so can be ignored.

[8] All of these calculations are available in a downloadable Excel file.

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value by the special zoning for the RHP project is: $11.46 − $5.65 = $5.81 million. This is the economic NPV of the Planning Commission's proposed Special Zoning Exemption.[9]

Furthermore, since the $11.46 million valuation of the RHP option is in fact the value of immediate exercise of that option, our model of option value is telling us that it is in fact optimal for the landowner to immediately begin construction on Phase I of the RHP project. Thus, the RHP project, like the as-of-right Rentleg Gardens project, is "*ripe*" for immediate development. This could be important for the Planning Commission to know, because for political reasons the town may wish to see the project commence construction as soon as possible. We see here one of the useful features of the option model of project valuation noted in Chapter 27, that it not only tells us the value of the project, but also whether it is currently optimal to begin immediate development.

In summary, the preceding real options value theory based analysis has included in a rigorous manner both the opportunity cost of capital and the value of flexibility and phasing in the possible development projects, in a model that is based fundamentally on the concept of market equilibrium across the markets for land, built property, and bonds. On this basis, the analysis allows us to make five important, policy-relevant conclusions:

1. A fair price for a "taking" of the Roth Harbor site based on its preexisting rights would be $5.65 million.
2. A fair bid for the site with the proposed special zoning exemption for the two-phase RHP Project would be $11.46 million.
3. Our best guess of the NPV of the Special Zoning Exemption is $5.81 million.
4. A recipient of the site with the special zoning exemption would likely seek to immediately begin construction on Phase I.
5. There is some urgency in closing an agreement with the current landowner, as the as-of-right (Rentleg Gardens) project is also ripe for immediate development (the current owner is suffering an opportunity cost by not proceeding with that development).

The option model also allows us to rigorously quantify the opportunity cost of capital for investment in the multiphase project as of any given state of the world, including the present (time 0). As described in Appendix 10C of Chapter 10, the OCC of the option can be "backed out" from the option valuation using the following procedure:

$$C_{i,j} = \frac{CEQ[C_{j+1}]}{1+r_f} = \frac{E_j[C_{j+1}]}{1+r_f+E[RP_{C_{i,j}}]} = \frac{E_j[C_{j+1}]}{1+OCC_{i,j}}, \Rightarrow 1+OCC_{i,j} = (1+r_f)\frac{E_j[C_{j+1}]}{CEQ[C_{j+1}]}$$

Thus, for the RHP project, the current OCC is given by:

$$\begin{aligned} 1+OCC_{0,0} &= (1+r_f)\frac{E_0[C_1]}{CEQ_0[C_1]} \\ &= 1.04\frac{(.786)15.71+(.214)1.20}{((.786)15.71+(.214)1.20)-(15.71-1.20)\left[\dfrac{9\%-4\%}{(1.15)-1/(1.15)}\right]} \\ &= 1.04\frac{\$12.60}{\$10.02} = 1.3085 \end{aligned}$$

In other words, the OCC for the RHP project is currently 30.85 percent.

[9] In this case, because of our simplification using relatively few, long periods (annual periods) in our binomial model (for illustrative purposes), the compound option value may be biased slightly on the low side, due to *"discreteness bias."* If we had constructed the model with shorter periods (a larger number of periods), we might have obtained a higher value for the option. (See Omberg, *JF*, June 1987: Unlike the case of simple options, the binomial model of compound options converges from below to the continuous time valuation.) However, it could be argued that in reality decision makers cannot actually make decisions continuously in time. If effectively the investment decision can actually only be practically reexamined on an annual basis, then our discretization models that reality.

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As described in section 29.1.2 of the printed text, this enables us to rigorously quantify the amount of investment risk in the RHP project, relative to that in the relevant built property assets. The following calculation:

$$\frac{E[RP_C]}{E[RP_V]} = \frac{30.85\% - 4\%}{9\% - 4\%} = \frac{26.85\%}{5.00\%} = 5.37$$

reveals that there is 5.37 times the risk (as perceived by the capital market) in the multiphase RHP development project as there is in an unlevered investment in completed apartment property investments.

Let us now consider how the multiphase RHP project would be evaluated using conventional development project valuation methods. Typically, a conventional DCF analysis would be applied:

- Ignoring the flexibility (but not ignoring the *expected* phasing) in the project;
- Using a nonrigorous *ad hoc* OCC as the discount rate.

Thus, a particular phasing scenario would be assumed. (Typically, the assumption would be that each phase will begin as soon as possible.) Then, a particular discount rate would be assumed. (In the early 2000s, a rate near 20 percent per annum has often been employed, in part (we suspect), because it is a nice round number and consistent with the "conventional wisdom" for required returns on development projects.)

In the case of our RHP project example, this conventional procedure would typically be applied as follows. First, consider that if both Phase I and Phase II are built as soon as possible, the net cash flow from the liquidation of each phase would be obtained in Years 2 and 4, respectively. Using the projected values of $E[V_2]$ and K_2, and of $E[V_4]$ and K_4, in Years 2 and 4 for Frenchman Cove and Fisher Landing, respectively,[10] we get the following net cash flow projection for the RHP project as a whole:[11]

Year:	1	2	3	4
	0	\$61.12 − \$49.94	0	\$103.76 − \$86.59
Net Cash Flow:		=\$11.18		=\$17.16

Discounting these net cash flows at the presumed OCC of 20 percent gives a gross present value for the RHP project of \$16.04 million:

$$PV = \frac{\$11.18}{1.20^2} + \frac{\$17.16}{1.20^4} = \$16.04$$

Thus, the conventional DCF approach (with a typical *ad hoc* discount rate of 20 percent) suggests a bid price of \$16.04 million for the RHP site with the Special Zoning Exemption. This compares to \$11.46 million using the real options approach.

In this example, the conventional approach has substantially over-estimated the more rigorously estimated value, due primarily to: (i) Using a discount rate that is too small

[10]Phase I (Frenchman Cove) has a *time* 0 completed value of \$60 million and a construction cost of \$48 million. With OCC of 9% and a cash yield of 8%, the implied growth rate in completed project value is 1.09/1.08 − 1 = 0.93%/year, which gives a projected Year 2 completed value of: \$60 million × 1.0093^2 = \$61.12 million. With construction costs expected to grow at 2% per year, their projected Year 2 value is: \$48 million × 1.02^2 = \$49.94 million. Similar projections for Phase II (Fisher Landing) starting from Fisher Landing's current (time 0) values of \$100 million and \$80 million for project value and construction are (\$100 × 1.0093^4) = \$103.76 for project value, and (\$80 × 1.02^4) = \$86.59 for construction cost.

[11]Here we are simplifying by working with annual frequency cash flow projections, to facilitate illustration. (In the real world, monthly or quarterly projections would be more common for a project of this scale.) We are also simplifying by using the "canonical" assumption that all construction costs for each phase are paid at the completion date of that phase.

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(20% vs. OCC = 30.85%); and (ii) Assuming the most optimistic project schedule (both phases implemented, and each as soon as possible).

In general, the conventional approach may either over- or underestimate the project value, relative to the real options valuation. Our point is *not* that the conventional approach is systematically biased. While a systematic bias (in one direction) in the conventional approach *may* exist, well-functioning investment markets for land, built properties, and riskless debt instruments should cause the conventional practice to tend to get valuation about right on average. Otherwise, opportunities for "*super normal*" profit (excess investment returns) would be widespread. The fact that our real options model is based fundamentally on the elimination of super normal ("arbitrage") profit suggests that the options approach and the conventional approach should tend to agree *on average* (across projects and over time).

With this in mind, we would summarize the key considerations between the real options approach versus conventional valuation as follows:

- The options approach is more *rigorous,* based fundamentally on market equilibrium, and thereby provides a valuation that is less error-prone, more likely to be "more correct" in more individual instances (even if not on average across all projects). Through this rigor,
- The options approach provides a deeper understanding of (i) the sources of the project value; and (ii) the true nature of the project investment risk and return
- The conventional approach is based on *ad hoc* assumptions regarding project execution and OCC, ignoring important realities of the project such as its flexibility and the principal of equalized risk-adjusted return expectations across investments.
- Even if the conventional approach gives a correct answer in a given case (i.e., the same valuation as the options approach), there is no way in itself to know *whether* the valuation is correct, or *why* it is correct if it is correct (except by basing such judgment on the more rigorous options approach).

Finally, let us consider two alternative project valuation approaches that were presented in the Chapter 29 printed text. The first is the procedure described in section 29.1.1. The procedure presented there is "rigorous" in the same way that we have used this term to describe the real options method, that is, it is based on true OCC rates derived from asset market equilibrium conditions. Yet this procedure did not involve a binomial model or any other sort of real options model. In fact, as was noted in sections 29.1.2 and 29.2, the procedure suggested in section 29.1.1 is consistent with the option model but only under the condition that it is optimal to immediately commit to irreversible construction of the *entire* development project. (This was in fact the situation we posited for the FutureSpace Center example project we considered in Chapter 29 of the main text.)

Suppose we applied the section 29.1.1 procedure to the multiphase RHP project considered in this section, even though that project does not meet the precondition of single-phase construction ripe for committed development. What would be the nature of the resulting valuation error?

In contrast to the conventional procedure we just examined, which uses *net cash flows* and an *ad hoc* discount rate, the section 29.1.1 procedure uses *gross cash flows* and rigorous OCC discount rates. The expected gross values of the to-be-completed assets area discounted back to present at their OCC rate, and the construction costs are discounted back to present at their OCC rate, and the subtraction to determine the NPV is performed using the discounted time 0 dollars. In the case of the two-phase RHP project, this procedure can be expressed in the following formula:

$$PV = PV[V_2^I - K_2^I] + PV[V_4^{II} - K_4^{II}]$$
$$= \left(\frac{E[V_2^I]}{(1+r_V)^2} - \frac{K_2^I}{(1+r_f)^2}\right) + \left(\frac{E[V_4^{II}]}{(1+r_V)^4} - \frac{K_4^{II}}{(1+r_f)^4}\right)$$

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The relevant cash flow stream is shown in the table below:

Year:	1	2	3	4
		$E_0[V_2] = \$61.12$		$E_0[V_4] = \$103.76$
Net Cash Flow:	0	$K_2 = \$49.94$	0	$K_4 = \$86.59$

And the calculations implementing the section 29.1.1 procedure are:

$$PV = \left(\frac{\$61.12}{1.09^2} - \frac{\$49.94}{1.04^2}\right) + \left(\frac{\$103.76}{1.09^4} - \frac{\$86.59}{1.04^4}\right) = \$5.27 - \$0.52 = \$4.75$$

Thus, this procedure estimates a gross value of the RHP project of only \$4.75 million, substantially below the \$11.46 million that we estimated using the options approach.

In general, the gross value DCF technique of section 29.1.1 will naturally underestimate the value of a multiphase project, even when it is optimal to immediately begin construction on the first phase, because this approach ignores the flexibility that exists to delay or cancel the subsequent phases. This flexibility has value. To see this, note that the principal source of the lower valuation in this approach is the present value of Phase II (the Fisher Landing project), which actually has a negative present value as of time 0 in the calculations above. Of course, in reality, the second phase could not possibly have a *negative* present value, as it has a positive expected net value upon completion, unless the developer is irrevocably committed from the outset to build Phase II, regardless of what happens in the real estate market between time 0 and the time when the Phase II is scheduled to start. But in fact this is not the case. If two years from now the apartment market has fallen, such that Fisher Landing would be worth less than its construction cost, the developer would simply not undertake to commence construction on the project at that time.

In summary, although the OCCs and cash flow projections used in the section 29.1.1 procedure are well justified, this procedure implicitly assumes an irreversible commitment at time 0 to complete the entire project as scheduled. It thus ignores the flexibility that actually exists, and thereby systematically under-estimates the project present value (in this case, \$4.75 million versus \$11.46 million).

Now let us consider the *ad hoc* procedure that was suggested in section 29.4 for evaluating multiphase projects without using option theory. The suggestion there is to use each phase's canonical OCC (as defined in section 29.1.2) to discount the *net* cash flow of each phase back to time 0. This approach is suggested as an approximation of the more rigorous options approach, useful for analysts who do not wish to employ the more complete options approach described in the present section. (Perhaps we should call this procedure "*semi-rigorous*" rather than "*ad hoc*," because the discount rates it employs are not completely without basis in market equilibrium considerations.) The section 29.4 procedure would be applied to the RHP project as follows.

First, a time line for the completion of all phases of the project would be posited. If we assume the earliest feasible completion of the RHP project, this would envision the previously noted net cash flow of \$11.18 million from the completion of Phase I at the end of Year 2, and \$17.16 million from the completion of Phase II at the end of Year 4. Computing the development OCC for Phase I based on equation (4) as described in section 29.1.2, we obtain a value of $E[r_c]$ for Phase I of 45.65 percent.[12] Similarly, for Phase II, we obtain an OCC of

[12]For Frenchman Cove with its two-year construction period: $PV[V_T] = \$61.12/1.09^2 = \$51.44$ million; $PV[K_T] = \$49.94/1.04^2 = \$46.17$; $NPV_0 = \$51.44 - \$46.17 = \$5.27$ million. Applying equation (4) and solving for $E[r_c]$, we obtain:

$$1 + E[r_C] = \left(\frac{V_T - K_T}{NPV_0}\right)^{1/T} = \left(\frac{\$61.12 - \$49.94}{\$5.27}\right)^{1/2} = 1.4565$$

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53.67 percent per year as of its projected construction start date at the end of Year 2.[13] Discounting each phase's projected *net* profit back to time 0 from their projected completion dates using each phase's computed OCC, we obtain a present value for both phases as of time 0 of \$8.35 million:

$$PV = \frac{\$11.18}{1.4565^2} + \frac{\$17.16}{1.5367^4} = \$8.35$$

This compares to the more rigorous and complete option-based valuation of \$11.46 million. Thus, in this case, our *semi-rigorous* procedure understates the true project value, in essence, by failing to account for the value of flexibility in the project (in particular, the flexibility to delay or abandon the second phase of the project). However, the *semi-rigorous* procedure gets closer to the true value than the gross value discounting methodology of section 29.1.1, while avoiding the completely *ad hoc* discounting of the conventional approach.

[13]For Fisher Landing with its two-year construction period, as of the earliest time it could be started (end of Year 2), we have: $PV[V_T] = \$103.76/1.09^2 = \$87.33$ million; $PV[K_T] = \$86.59/1.04^2 = \$80.06$; NPV_0 \$87.33 − \$80.06 = \$7.27 million. Applying equation (4) and solving for $E[r_c]$, we obtain (with round-off):

$$1 + E[r_C] = \left(\frac{V_T - K_T}{NPV_0}\right)^{1/T} = \left(\frac{\$103.76 - \$86.59}{\$7.27}\right)^{1/2} = 1.5367$$

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