APPENDIX 10C

THE CERTAINTY EQUIVALENCE APPROACH TO DCF VALUATION

The DCF method presented in the body of this chapter is the traditional *risk-adjusted discounting* approach, in which risky future cash flow or value amounts are discounted to present value using a risk-adjusted discount rate (RADR) that reflects the opportunity cost of capital (OCC) for investments of similar risk to that of the future claim being discounted. While this traditional approach is by far the most widely employed in practice, it is possible to define a different approach that is equivalent, but provides a useful additional perspective. This alternative approach is often called *certainty equivalence valuation,* and it is presented in most graduate school finance textbooks. The certainty equivalence approach provides some capabilities that the traditional approach does not. In particular, we will see later in this book how certainty equivalence valuation will enable the rigorous valuation of projects that are characterized by significant flexibility or “optionality,” such as many large-scale real estate development projects. Such projects cannot be evaluated using traditional risk-adjusted discounting, because it is impossible to know what the correct OCC to apply to the project would be. The certainty equivalence approach can often shortcut the need to know the OCC, while actually providing a means to discover what the true OCC is.[1] In this appendix, we will introduce the certainty equivalence approach and provide a brief example of what we mean.

Consider the basic element in the traditional risk-adjusted discounting approach to DCF valuation, the discounting to present value of a single expected future value that will be obtained one period in the future, using a risk-adjusted discount rate expressed as a simple periodic return expectation:

$$PV[V_1] = \frac{E_0[V_1]}{1 + E_0[r_V]}$$

Here we are accounting for both time and risk in the discount rate in the denominator, as $E_0[r_V] = r_f + RP_V$, where r_f is riskless and accounts for the time value of money, and RP_V is the market’s required risk premium in the expected total return for the investment. But we can easily expand and algebraically manipulate this formula so that the denominator purely reflects the time value of money (the discounting is done *risklessly*) and the risk is completely and purely accounted for in the numerator:

$$\begin{aligned} PV[V_1] &= \frac{E_0[V_1]}{1 + E_0[r_V]} = \frac{E_0[V_1]}{1 + r_f + RP_V} \\ (1 + r_f + RP_V)PV[V_1] &= E_0[V_1] \\ (1 + r_f)PV[V_1] + (RP_V)PV[V_1] &= E_0[V_1] \\ PV[V_1] &= \frac{E_0[V_1] - (RP_V)PV[V_1]}{1 + r_f} = \frac{CEQ_0[V_1]}{1 + r_f} \end{aligned} \qquad (C1)$$

[1]Another use of the certainty equivalence method is in the valuation of certain real estate derivatives that are futures contracts, such as index return swaps of the type presented in Chapter 26. In a futures contract no cash changes hands up front, implying an equilibrium present value of zero. Obviously, no finite risk-adjusted discount rate can discount non-zero future expected cash flows to a present value of zero. But the certainty equivalence approach can handle such valuation problems.

The future value in the numerator on the right-hand side, labeled $CEQ_0[V_1]$, is referred to as the **certainty equivalent value**. Notice that the certainty equivalent value equals the (unbiased) expected value in dollars, $E_0[V_1]$, less a "risk discount" expressed in dollars, $(RP_V)PV[V_1]$. Numerically, the risk discount equals the risk premium component of the OCC (which is a decimal or percent) times the present value of the claim in dollars. Conceptually, the certainty equivalent value is the amount such that the investment market would be indifferent between a riskless claim to receive that amount for certain, and the actual claim to receive the risky amount V_1 which could turn out to be either greater or less than $E_0[V_1]$ (given that the expectation is unbiased). Thus, the appropriate OCC to use in discounting the $CEQ_0[V_1]$ value is the risk-free rate, r_f.

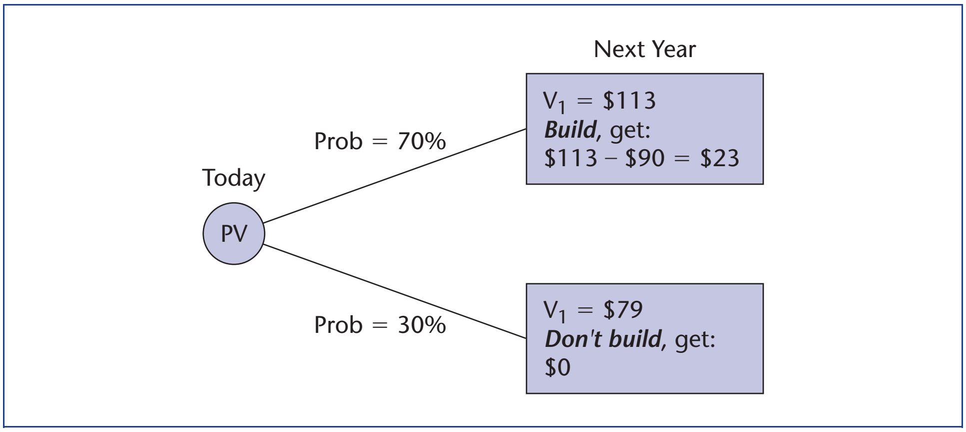
To see how this can work in practice, let's consider a simple numerical example. Suppose that the claim we are evaluating is an office building that will be worth next year either \$113 million (with a 70 percent probability) or \$79 million (with a 30 percent probability). Suppose further that our office building does not yet exist, but we could build it next year for a construction cost of \$90 million. The question is, what is the present value of our option (that is, our "right without obligation") to build the office building next year.

Exhibit 10C-1 depicts the situation we face. If the "*up*" outcome materializes, we will build the office building and obtain an asset worth \$113 million for a construction cost of \$90 million, providing a net profit of \$23 million. If the "*down*" outcome materializes, we will, of course, not build the office building, saving our \$90 million of construction cost, since that would only produce an asset worth \$79 million (thereby avoiding an obviously *negative NPV* decision), leaving us with zero.[2]

To evaluate the option, suppose first that we can ascertain that the present value of a claim on the office building in question a year from now would have a present value today of \$94 million.[3] Or, equivalently, suppose we can observe that office buildings like this have an OCC of 9 percent, that is, they command an expected return of 9 percent in the asset market. And suppose that the risk-free interest rate (e.g., interest rate on government bonds) is 3 percent. This situation is depicted in the equation below:

$$PV[V_1] = \frac{E_0[V_1]}{1 + E_0[r_V]} = \frac{pV_1^{up} + (1-p)V_1^{down}}{1 + r_f + RP_V} = \frac{(0.7)\$113 + (0.3)\$79}{1 + 0.03 + 0.06} = \frac{\$103}{1 + 0.09} = \$94$$

EXHIBIT 10C-1 Binomial Outcome Possibilities



[2]To simplify the illustration, we will assume that construction is instantaneous, that there are no further future periods of time (i.e., our "option" on the land expires after one year), and that the only thing we could do with the land is to build the subject office building.

[3]An identical office building *already existing* today might be worth \$100 million. But this would include about \$6 million of present value of the expected net rental income the pre-existing building would generate between now and next year, income that our not-yet-existing (to-be-built) building cannot provide, as it will not exist until it is constructed next year. (This assumes prevailing cap rates, or net income yields, for office buildings are about 6 percent.)

Note that if we know the expected future value (103) or the future scenario for the office building (the 70 percent probability of \$113 and 30 percent probability of \$79), then it suffices to know *either* the OCC (in this case 9 percent) *or* the current value (in this case \$94) of the investment asset in order to determine the other variable (just solve the above equation for the unknown variable), and thereby to completely determine the investment present value and expected return question. As at least one of these parameters will usually be at least somewhat observable empirically in the investment asset marketplace, it is relatively easy to evaluate assets such as the future claim on the office building using risk-adjusted discounting. But it is much more difficult to directly empirically observe either the OCC or the present value of the *option* to build the office building as we have described it.

But let us suppose (rather plausibly) that the market's required risk premium in an investment, *RP*, is proportional to the "risk" in the investment as measured by the percentage spread in the possible change in value of the investment between now and next year. In the case of the office building, this percentage spread is the difference between the +20% rise in value (to \$113 from \$94) in the "*up*" outcome, and the −17% fall in value (to \$79 from \$94) in the "*down*" outcome. This outcome spread of 37 percent measures the amount of "risk" in an investment in the office building. Given the office building's required risk premium of 6 percent (equal to its 9 percent OCC minus the 3 percent risk-free interest rate), this implies that the market's "*risk premium per unit of risk*" is 6% divided by 37%, or 0.162. This is effectively the investment market's "price of risk" for this type of asset. This means that the following relationship, which elaborates on the structure we have just described, must hold:

$$\begin{aligned}\frac{9\% - 3\%}{(\$113 - \$79)/\$94} &= \frac{RP_V}{(V_1^{up}\$ - V_1^{down}\$)/PV[V_1]} = \frac{RP_V}{V_1^{up}\% - V_1^{down}\%} \\ &= \frac{9\% - 3\%}{(+20\%) - (-17\%)} = \frac{6\%}{37\%} = 0.162\end{aligned} \qquad (C2)$$

Of course, the same relationship must also hold for the option, if the market is to be in equilibrium. In other words, the same "price of risk" applies to all assets, and their required return risk premia must all have the same proportion to their risk. Labeling the present value of our option as $PV[C_1]$, and recalling from Exhibit 10C-1 that the option will be worth either \$23 in the "*up*" outcome or zero in the "*down*" outcome, we must have:

$$\frac{RP_C}{C_1^{up}\% - C_1^{down}\%} = \frac{RP_C}{(C_1^{up}\$ - C_1^{down}\$)/PV[C_1]} = \frac{RP_C}{(\$23 - \$0)/PV[C_1]} = 0.162 \qquad (C3)$$

The trouble is that, as we cannot readily observe from the market the value of either $PV[C1]$ or of RP_C, we cannot use risk-adjusted discounting to evaluate the option. However, the above equation does allow us to evaluate $(RP_C)PV[C_1]$, which, using certainty equivalence valuation, is all we need in order to evaluate the option.[4]

From equation (C3), we have:

$$\begin{aligned}&\frac{RP_C}{(C_1^{up}\$ - C_1^{down}\$)/PV[C_1]} = 0.162 \\ &\Rightarrow RP_C = 0.162(C_1^{up}\$ - C_1^{down}\$)/PV[C_1] \\ &\Rightarrow (RP_C)PV[C_1] = 0.162(C_1^{up}\$ - C_1^{down}\$) = 0.162(\$23 - \$0) = \$3.73\end{aligned} \qquad (C4)$$

[4]The approach described here has been developed by Tom Arnold and Timothy Crack. For further elaboration, see T. Arnold and T. Crack, "Option Pricing in the Real World: A Generalized Binomial Model with Applications to Real Options," Department of Finance, University of Richmond, Working Paper, April 15, 2003. As will be elaborated in Chapter 27, we are here using the fact that the development option is a "derivative" of the built office building, that is, the development project's outcome next year is perfectly correlated with the outcome for the built office building. Thus, the option and the building have the same "type" of risk (only a different amount of it).

From equation (C1), the certainty equivalence valuation formula, we have (for the option):

$$PV[C_1] = \frac{CEQ_0[C_1]}{1+r_f} = \frac{E_0[C_1] - (RP_C)PV[C_1]}{1+r_f} = \frac{[(0.7)\$23 + (0.3)0] - (RP_C)PV[C_1]}{1+0.03}$$
$$= \frac{\$16.1 - (RP_C)PV[C_1]}{1.03} \qquad (C5)$$

Substituting from (C4) into (C5), we obtain the value of the option:

$$PV[C_1] = \frac{\$16.1 - (RP_C)PV[C_1]}{1.03} = \frac{\$16.1 - \$3.73}{1.03} = \frac{\$12.37}{1.03} = \$12 \qquad (C6)$$

The option is worth \$12 million today.

Expanding and rewriting (C6) so as to combine the relevant elements of the preceding equations, we arrive at the general certainty equivalence valuation formula for a single period binomial world:

$$PV[C_1] = \frac{E_0[C_1] - (C_1^{up}\$ - C_1^{down}\$)\dfrac{E_0[r_V] - r_f}{V_1^{up}\% - V_1^{down}\%}}{1+r_f}$$
$$= \frac{\$16.1 - (\$23 - 0)\dfrac{9\% - 3\%}{20\% - (-17\%)}}{1.03} = \frac{\$16.1 - \$23\left(\dfrac{6\%}{37\%}\right)}{1.03} = \frac{\$12.37}{1.03} = \$12 \qquad (C7)$$

Having obtained the present value of the option, we can of course now "back out" the OCC and the risk premium for the option:

$$PV[C_1] = \frac{E_0[C_1]}{1+E_0[r_C]} = \frac{\$16.1}{1+E_0[r_C]} = \$12$$
$$\Rightarrow E_0[r_C] = \frac{\$16.1}{\$12} - 1 = 0.34$$

The OCC for the option is 34 percent, which means that its expected return risk premium is: $RP_C = E_0[r_C] - r_f = 34\% - 3\% = 31\%$. This suggests that the option has $RP_C/RP_V = 31\%/6\% \approx 5$ times the investment risk of an investment in pre-existing office building like the one that the option allows us to build. The risk magnification derives from the "leverage" that is inherent in the fact that the development project requires an investment of an essentially riskless amount of \$90 million in construction cost in order to obtain the office building.[5]

[5]The key point is not that the construction costs are necessarily fixed in advance (though they may be), but rather that they are not much positively correlated with either the value outcome of the office building or the value outcomes of other major types of financial assets. This lack of correlation is sufficient to produce the leverage.