APPENDIX 30B

NUMERICAL EXAMPLE OF LANDLORD LEASE TERM INDIFFERENCE RENT

We noted in section 30.4.1 in the text that consideration of interlease risk typically makes the landlord prefer longer-term leases, other things being equal (such as in particular the lease rent per year). The implication of this is that the landlord will be indifferent between longer-term leases with lower effective rents and shorter-term leases with higher effective rents. For the same underlying reason, tenants will typically have the opposite term preference based on interlease risk, preferring shorter-term leases, other things being equal. This results in tenants having the same general downward-sloping indifference rent. That is, tenants typically will be willing to pay higher rents for shorter-term leases, at least in the face of flat future market rent projections. Together, these two results imply that the equilibrium or normative term structure of rents will be downward-sloping relative to the projection of future spot market rents.[1] This general relationship, and the typical magnitude of the implied indifference rent slope, will be demonstrated with a simple numerical example in this appendix.

Suppose that the rental market is characterized as follows:

- The intralease discount rate is 8 percent for both the landlord and the tenant.
- The interlease discount rate is 12 percent for both the landlord and the tenant.
- Spot rents (short-term leases) are expected to remain flat at \$100/year, net.
- There are no releasing costs and there is no vacancy between leases.

Under these assumptions, the building value to the landlord is a perpetuity of the expected future rental payments.

Now consider the building value assuming short-term rental, that is, with the building entirely exposed to the spot rental market every year. In this case, all expected cash flows are discounted at the interlease discount rate because there are no long-term leases locking in contractually fixed rents. Thus, the building value (V) is \$833.33, calculated as follows:

$$V = \frac{\left[\frac{1.08}{0.08}\left(1-\left(\frac{1}{1.08}\right)^{1}\right)\$100\right]\Big/ 1.12}{1-\left(\frac{1}{1.12}\right)^{1}} = \frac{\$100}{0.012} = \$833.33 \tag{B.1}$$

In formula (B.1), we have expanded the simple perpetuity formula as an infinite series of one-period annuities to illustrate that we are evaluating the building as a series of short-term

[1]The analysis presented here is essentially consistent with the early, seminal literature on the term structure of general lease rents, such as Miller and Upton (1976), McConnell and Schallenheim (1983), and Schallenheim and McConnell (1985). The fundamental equilibrium condition is that the present value of the lease must equal the present value of the asset's service flow. Here we interpret the present value of the asset's service flow as the market value of the building. This equilibrium condition also underlies more recent contingent claims based models, such as Grenadier (1995b and 2005) and Clapham and Gunnelin (2003). To focus purely on the effect of lease term (independent of rental market directions), we assume, in this appendix, a flat future rental market.

leases, within each of which the intralease discount rate of 8 percent may be applied.[2] For simplicity, we assume that rents are paid annually in advance, with the first rent payment to be received at the end of the first year when the first lease will be signed (that is, "ex-dividend" current valuation, as if this is a new building with one year expected until the first rental payment).

Next consider the same building value assuming that the space will be perpetually released once every 10 years in successive 10-year long-term leases made at the spot market rate each time (of course, with the same assumptions as before regarding rent paid in advance and the first lease signed one year from present). In this case, the value of the building would be \$954.30, computed as in formula (B.2).[3]

$$V = \frac{\left[\frac{1.08}{0.08}\left(1-\left(\frac{1}{1.08}\right)^{10}\right)\$100\right] \Big/ 1.12}{1-\left(\frac{1}{1.12}\right)^{10}} = \$954.30 \tag{B.2}$$

Note that the expected effective rent in the 10-year leases in (B.2) is the same as that in the one-year leases (spot rent) in (B.1), namely, \$100.[4] Yet we have a higher building value in (B.2). It appears that the landlord can increase property value simply by signing longer-term leases, provided she can get the same rent per year. Mechanically, the increased use of the intralease discount rate in the building perpetuity valuation, reflecting the reduced cash flow uncertainty due to the use of long-term leases, allows a 14.52 percent increase in building value in this case, from \$833.33 to \$954.30.

Another way of putting this is to say that, all else being equal and assuming constant projected future spot rents, landlords should be indifferent between shorter-term leases at higher rents and longer-term leases at lower rents. In particular, in the previous example (with an 8 percent intralease discount rate and a 12 percent interlease discount rate), rents for 10-year leases could be $1/1.1452 = 87.32$ percent of the short-term spot rental rate, and the landlord would then be indifferent between either a one-year lease and a 10-year lease because either type of lease would give the same property value. If spot rents are \$100, then this indifference on the part of landlords would occur if 10-year lease rents were \$87.32 per year, as confirmed in formula (B.3):

$$V = \frac{\left[\frac{1.08}{0.08}\left(1-\left(\frac{1}{1.08}\right)^{10}\right)\$87.32\right] \Big/ 1.12}{1-\left(\frac{1}{1.12}\right)^{10}} = \$833.33 \tag{B.3}$$

Alternatively, if effective rents on 10-year leases were indeed \$100, then the spot rent on short-term leases would be \$114.52 for landlord indifference. In general, if future spot rents are projected to remain constant at the current level, then the indifference rent will assume a downward-sloping curve as a function of the lease term, as indicated in the chart

[2]You should recognize the expression in the square brackets in the numerator of (B.1) as the present value of a level annuity with payments in advance ($n = 1$, $r = 8$ percent). This value is received every $T = 1$ years in perpetuity, starting one year from the present. So the rest of formula (B.1) is the level infinite geometric series formula, discounted to time *0*. The overall numerator is the value of the first term in the series (the value of the first lease, as of time *0*). The denominator on the bottom is one minus the common factor discounting each subsequent lease: ($1/[1 + r]^T$), where r is the interlease discount rate, and T is the length of each lease term (in this case, 12 percent and 1, respectively).

[3]Mathematically, formula (B.2) is the same formula as formula (B.1), the level annuity formula imbedded in the level perpetuity formula, discounted one period to time *0*. The only difference is that in (B.2) the lease term, T, is 10 years instead of one. (See section 8.2.7 in Chapter 8.)

[4]This is consistent with the flat rent expectations in the space market.

EXHIBIT 30B-1
Indifference Rent as a Function of Lease Term (due to interlease risk impact only, assuming flat spot rent expectations)

in Exhibit 30B-1 (which assumes an intralease discount rate of 8 percent and an interlease discount rate of 12 percent).[5]

Now consider the tenant's perspective on this same issue. The key point is that, assuming the same intralease and interlease discount rates, the absolute present value of a perpetual stream of rent payments is the same to the tenant as it is to the landlord, only this cash flow stream is a cost to the tenant whereas it is a positive value to the landlord. In other words, the tenant cash flow stream is just the negative of the landlord's cash flow stream. Thus, tenants have the same downward-sloping lease term indifference rent curve as landlords do (with constant spot rents).

An example may clarify the intuition behind this result. Suppose the tenant uses the space in question to produce widgets that are sold for $1 each with a variable production cost of $0.50 each. Expected production is 1,000 widgets per year in perpetuity. The opportunity cost of capital for widget production investment (apart from rent) is 10 percent per year. If the rent is $100/year, then the value of the tenant firm is

$$
\begin{aligned}
V &= PV(\textit{widget net income}) - PV(\textit{rent}) \\
&= \$500/1.10 - PV(\text{rent}) \\
&= \$5{,}000 - \$833 = \$4{,}167, \text{ if 1-year leases at } 100/\text{year} \\
&= \$5{,}000 - \$954 = \$4{,}046, \text{ if 10-year leases at } 100/\text{year}
\end{aligned}
$$

Thus, the tenant prefers short-term leases.

As a result, the equilibrium rent term structure that would allow both landlords and tenants to be indifferent across leases of different term lengths is downward-sloping. Tenant firm value equals:[6]

$$
\begin{aligned}
V &= \$5{,}000 - \$833 = \$4{,}167, \text{ if 1-year leases at } 100/\text{year} \\
&= \$5{,}000 - \$833 = \$4{,}167, \text{ if 10-year leases at } 87.32/\text{year}
\end{aligned}
$$

[5]Note that the intralease and interlease discount rates are not generally based on the same point in the bond market yield curve. In particular, the intralease discount rate is based on the yield of bonds with the same duration as the lease cash flows, while the interlease discount rate is based on the yield of bonds with duration at least equal to the lease term. As leases are characterized by level payments in advance, the lease duration is substantially shorter than the lease term. Thus, if the bond market yield curve has its typical upward-sloping shape, then this is another reason (in addition to space market risk) that the interlease discount rate is typically higher than the intralease discount rate.

[6]Again, this assumes constant future projected spot rents. If future spot rents are not constant, the declining indifference curve refers to rent *relative to* what it would otherwise be, reflecting the expected future spot rents as well as the impact of rental market risk.